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Experiment and m.h.d. Theory of Stability and Relaxation in Toroidal Discharges

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Experiment and m.h.d. theory of stability and relaxation in toroidal discharges

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The experimental behaviour of plasma instabilities in high-current discharges is found to be in good agreement with the predictions of linear and nonlinear magnetohydrodynamic theory. Observations show that on time-scales comparable with the Alfvèn transit time there are rapidly growing ideal magnetohydrodynamic perturbations whereas experiments on longer time-scales show the growth and saturation of resistive instabilities which involve changes in field line topology. The plasmas are observed to exhibit self-control mechanisms which are related to the relaxation of configurations to states of lower magnetic energy. Rapid magnetic field line reconnection phenomena, as in solar flares, are observed.

Introduction

Since the earliest attempts to produce the conditions needed for controlled thermonuclear fusion large-scale instabilities have been a major obstacle to progress, as they limit the current and pressure that can be confined by a magnetic field. However the most active instabilities can be suppressed by means of rigid conductors and currents and fields induced by external sources. It is now known both from experiment and theory that the addition of a longitudinal field (B_ϕ) can give equilibria which are stable to almost all perturbations. Two examples are the tokamak and the reverse field pinch.

If high-current discharges are to be used for containing the plasma needed for a controlled thermonuclear reactor, we need to know much detail about the instabilities, as small fluctuations and oscillations are observed in most of the fusion-relevant plasmas produced to date. In contrast, quiescent plasmas appear to be relatively cold and dominated by dissipative processes and radiation. When the disturbances grow to large amplitude (typically 1–10% of the equilibrium values) we need to know which instabilities can be tolerated and which are to be avoided. For example, a relaxation oscillation associated with a growing perturbation that subsequently redistributes current and pressure in the hot core of a plasma is relatively harmless.

Most magnetically confined plasmas in the laboratory display self-control mechanisms, rather than the explosive onset observed in solar flares and similar astrophysical phenomena, which appear to be associated with the rapid reconnection of magnetic field lines. The self-control mechanism can be related to the general principle that the magnetic field lines will reconnect and cause the configuration to relax to a final state of lower magnetic energy subject to a small number of constraints.

This paper summarizes the comparisons that have been made between theoretical predictions and observations principally made on two toroidal discharge assemblies, H.B.T.X.1 – a reverse field pinch (Verhage 1978) – and Tosca – a small tokamak (Cima 1977). It is concluded that the properties of growing perturbations can be predicted by theory to within the accuracy of experiment (± 10%). Many of the qualitative features of the nonlinear phase can also be predicted if a single or small number of instabilities are involved. Relaxation theory satisfactorily describes many of the general features of the evolution of a toroidal discharge, particularly if a large number of perturbations are involved.

Ideal m.h.d. instabilities

M.h.d. instabilities manifested themselves in the earliest attempts at controlled thermonuclear fusion by using toroidal pinches. If the experimental time-scale is comparable with the transit time for an Alfvèn wave ($\tau_A = a/v_A$, where $v_A^2 = B^2/\mu_0\rho$, a is the tube radius, B is the magnetic field and ρ is the mass density) then it is possible to create pinches where most of the plasma current is longitudinal (z-pinch) or where the current is poloidal (θ-pinch). However on longer time-scales a variety of instabilities are observed.

These ideal instabilities can be investigated theoretically by solving the equations of motion and state for a perfectly conducting fluid:

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\nabla p + \boldsymbol{J} \wedge \boldsymbol{B}; \quad \frac{\partial \boldsymbol{B}}{\partial t} = \nabla \wedge (\boldsymbol{v} \wedge \boldsymbol{B}); \quad \frac{\mathrm{d}(p\rho^{-\gamma})}{\mathrm{d}t} = 0. \tag{1}$$

The method of solution is to find the eigenvalue ω and displacement $\xi(\boldsymbol{r}, t)$ which, for example, in cylindrical geometry takes the form

$$\xi(\boldsymbol{r}, t) = \xi(r)\, \mathrm{e}^{\omega t + \mathrm{i}(m\theta + kz)},$$

where m is the poloidal mode number and k the axial wavenumber. The eigenvalue equations are well suited to computational evaluation and detailed verification of the growth and structure of observed m.h.d. instabilities has been made in many cases though here we consider a particular example. (It is often more convenient for marginal stability investigations to use a minimization of the potential energy (Newcomb 1960). This later approach, however, does not give the growth rates or the eigenfunctions.) Figure 1 shows a measured field configuration from a toroidal pinch (Verhage 1978) at the instant just before the plasma becomes unstable. Figure 2 shows the measured perturbation in the poloidal field B_θ and the toroidal or axial field B_ϕ, and the radial field b_r, during the growth of the $m = 1$ kink instability. The predicted perturbations for the most rapidly growing mode are also shown in figure 2. The helical perturbation matches the pitch of the magnetic field at a radius of about 17 mm. It is a direct consequence of ideal m.h.d. theory that the radial field component changes sign at this radius and that the topology of the magnetic surfaces is not changed by the instability but merely displaced until the plasma interacts with the boundary. The inclusion of the appropriate values of resistivity, viscosity and thermal conductivity, associated with the measured plasma parameters, does not alter the calculated field perturbations, which shows that the instability is adequately described by ideal m.h.d. theory. The agreement between computation and measurement is good to within ± 10% for the perturbations, the axial wavelength and the growth rate; the agreement for the B_ϕ-perturbation at small radius is poor but this can be accounted for by including nonlinear effects of the helical instability, which converts poloidal flux into toroidal flux.

The radial variation of the pitch of the magnetic field plays a crucial role in the stability of a toroidal confinement system to m.h.d. perturbations. In particular, it is necessary to avoid a minimum in the profile of pitch against distance from the magnetic axis. In tokamak devices the normalized pitch, or safety factor, $q = r B_\phi / R B_\theta$, where R is the major radius, increases continuously with distance from the magnetic axis (figure 3*a*). In devices with a poloidal field comparable with the toroidal or axial magnetic field B_ϕ, avoidance of a pitch minimum requires

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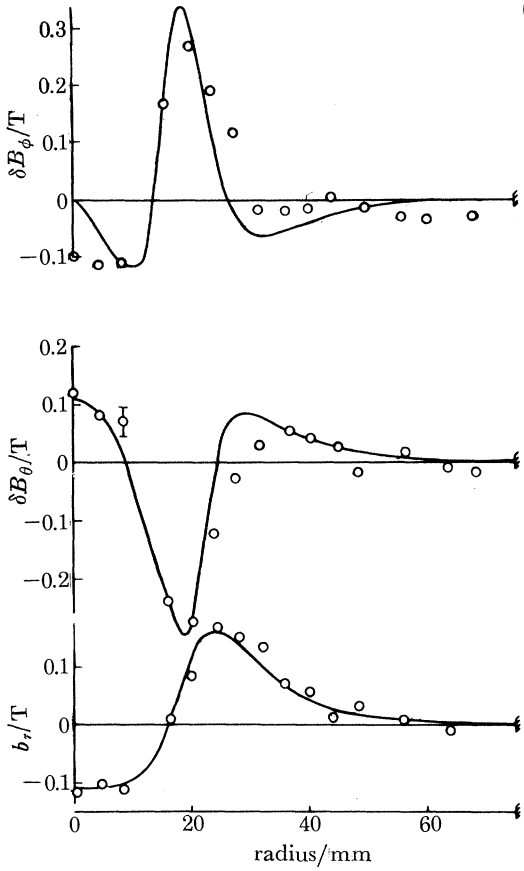

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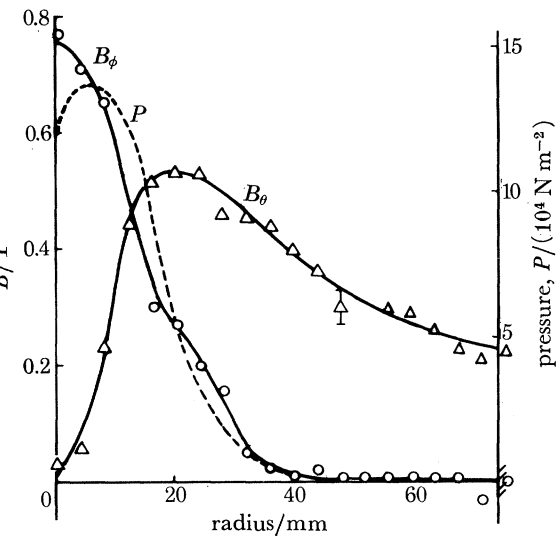

FIGURE 1. Measured field configuration at $t = 5$ μs, $\beta_\theta = 0.59$. B_ϕ is the axial or toroidal field and B_θ the poloidal field. – – –, Pressure profile derived from the equilibrium relation.

FIGURE 2. Measured and predicted magnetic field perturbations associated with the kink instability at $t = 5.8$ μs for the configuration shown in figure 1.

that the pitch change sign between the magnetic axis and the plasma boundary (figure 3*b*). This can be achieved by reversing the sign of the axial field B_ϕ, which is the basis of the reverse field pinch confinement concept (Rosenbluth 1958; Bodin 1975).

Resistive instabilities

The addition of resistivity to the ideal equations (1) reduces the growth rate of ideal m.h.d. instabilities due to the dissipation of electric currents; however, it can produce new instabilities by removing constraints from the equations and by making lower potential energy states accessible to the plasma (Furth 1963). In a fluid with infinite electrical conductivity the topology of the magnetic lines of force is invariant, the lines moving with the fluid. In the presence of resistivity the topology is no longer invariant and field lines may break and reconnect. The most important of these instabilities is perhaps the resistive tearing mode. This mode involves the break-up of the field in the neighbourhood of the neutral line between oppositely directed field components in a thin boundary layer within the plasma. Resistive tearing modes grow on a time-scale slower than τ_A but faster than the typical diffusion time-scale ($\tau_\sigma = \mu_0 \sigma a^2$, where

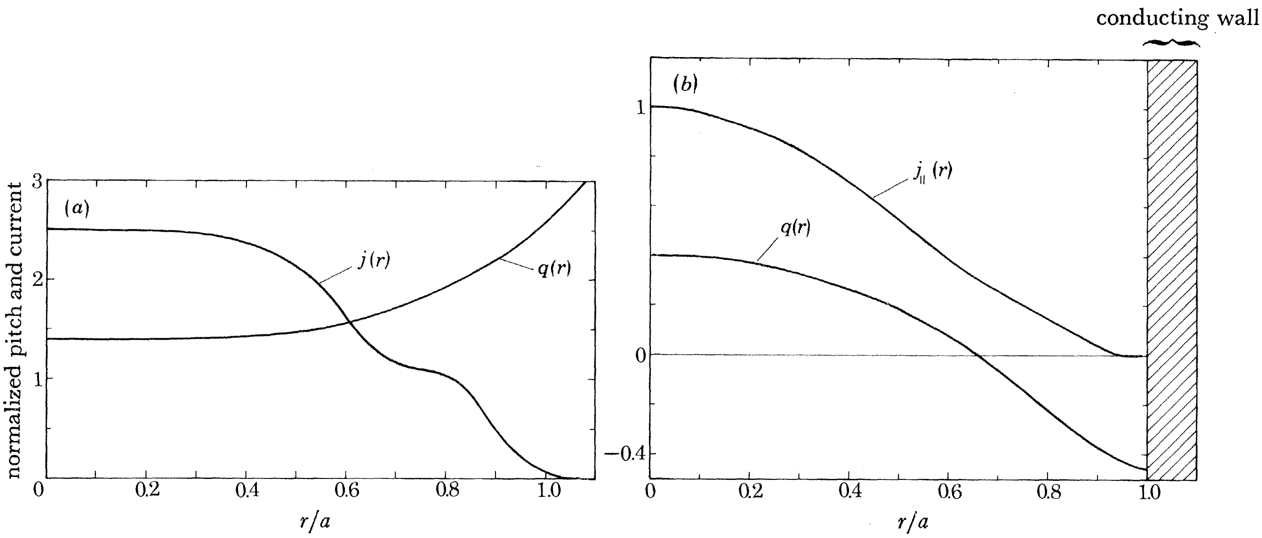

FIGURE 3. (*a*) Stable current distribution for a tokamak, showing the radial variation of the normalized pitch, $q(r)$. (*b*) Stable parallel current distribution for a reverse field pinch showing the radial variation of the normalized pitch. Here, a conducting wall, whose position is indicated, is required to stabilize the pinch.

σ is the electrical conductivity) and are driven by gradients in the current density. Resistive instabilities in cylindrical geometry lead to magnetic surfaces that have helical magnetic island structures (an example is shown in figure 4 for poloidal mode number $m = 1$) and a radial magnetic field perturbation that does not change sign with distance from the magnetic axis (unlike the 'ideal' perturbation shown in figure 2).

Resistive instabilities can be investigated computationally and compared with experiment by solving the equations of motion and state with resistivity, viscosity and thermal conductivity included, as an initial value problem (Killeen 1970). The critical parameter is the Lundquist number (or magnetic Reynolds number) $S = \tau_\sigma/\tau_A$. If $S \approx 10^2$ then dissipation has a marked effect on ideal m.h.d. instabilities which limits the spectrum of modes that can appear. When $S \gtrsim 10^3$ the boundary-layer thickness ($\propto S^{-\frac{1}{3}}$) becomes a small fraction of the plasma radius, dissipation effects are no longer very important and resistive instabilities may grow rapidly, for example on a time-scale of $\tau_A/S^{-\frac{1}{3}}$.

Experiments and nonlinear calculations show that the magnetic islands associated with the $m = 1$ tearing mode exponentiate to a size comparable with the radius, which ultimately leads to a new set of nested flux surfaces as shown in the time sequence of figure 4.

A second important resistive instability is the resistive interchange mode driven by the pressure gradient in a curved magnetic field (Furth 1963). These instabilities grow more rapidly than tearing modes and again produce magnetic islands. This mode and the resistive tearing mode have been detected in pinch experiments (Carolan 1979).

Resistive stability theory indicates that there are a range of current distributions for the reverse field pinch and tokamak that are stable to the resistive tearing mode, in the first case owing to wall stabilization and in the second owing to reduced current gradient in the region of the most unstable modes. An example of each, with the parallel current zero at the edge of the plasma, is shown in figure 3 with the respective variations of the pitch.

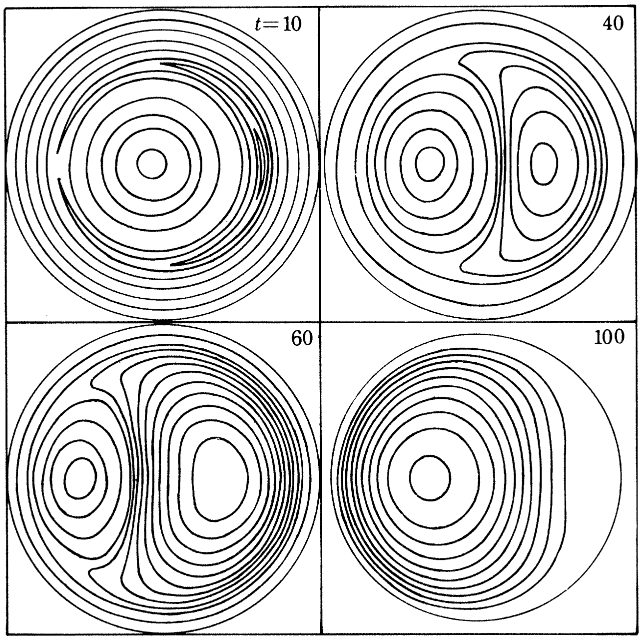

FIGURE 4. Evolution of the magnetic surfaces for a growing resistive tearing mode in a reverse field pinch, from a nonlinear calculation. The growth of the magnetic island into a new magnetic axis can be seen.

INSTABILITIES IN TOKAMAKS

In a slow sustained pinch device such as a tokamak (figure 3) in which the pulse length may be many millions of Alfvèn transit times, rapid instabilities characteristic of ideal m.h.d. are not usually observed. Here the pitch length of the field lines is everywhere greater than the distance around the torus ($q > 1$), and the plasma evolves in such a manner that although the slower and weaker resistive instabilities may appear they act as self-control mechanisms on the current distribution which determines the stability.

Two forms of benign resistive m.h.d. instabilities are often observed during normal tokamak operation which are in close agreement with the detailed predictions of m.h.d. theory. 'Sawtooth' oscillations are observed as fluctuations in the soft X-rays emitted from the central hot core of the plasma (figure 5*a*) (von Goeler 1974). In contrast, Mirnov oscillations (Mirnov 1971) are observed as regular magnetic field oscillations which can be detected outside the plasma column and correspond to moving helical structures near the edge of the plasma.

Sawtooth oscillations result from the current density tending to concentrate at the magnetic axis as a result of the higher temperature and hence lower resistivity there. This reduces the safety factor q to less than one; a resistive tearing mode with $m = 1$, toroidal mode number $n = 1$ ($k = n/R$), then grows rapidly and leads to a sudden drop in temperature and current at the centre of the plasma, figure 5*a*. The oscillations are believed to be associated with the rotation of the growing magnetic island in the direction of electron current flow, which is connected with the Hall effect in Ohm's law. The detailed structure of the $m = 1$ instability can be obtained from an array of soft X-ray detectors. Figure 5b compares the radial structure measured in this way with the theoretical predictions. Nonlinear calculations can accurately reproduce the observed sawtooth behaviour (Sykes 1976) and the flux surfaces evolve similarly to those shown in figure 4.

Measurements with magnetic probes and soft X-rays reveal that the Mirnov oscillations are rotating magnetic islands produced by nonlinearly saturated resistive tearing modes. Figure 6

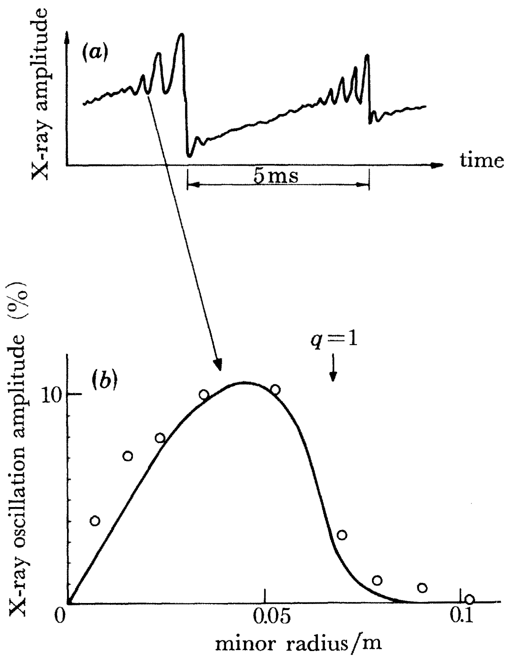

FIGURE 5. (a) Soft X-ray signal, from the central region of a tokamak plasma, showing sawtooth oscillations. (b) Radial profile of the oscillating X-ray component from the Princeton Large Torus (P.L.T.) device compared with the predicted profile for an $m = 1$ tearing mode.

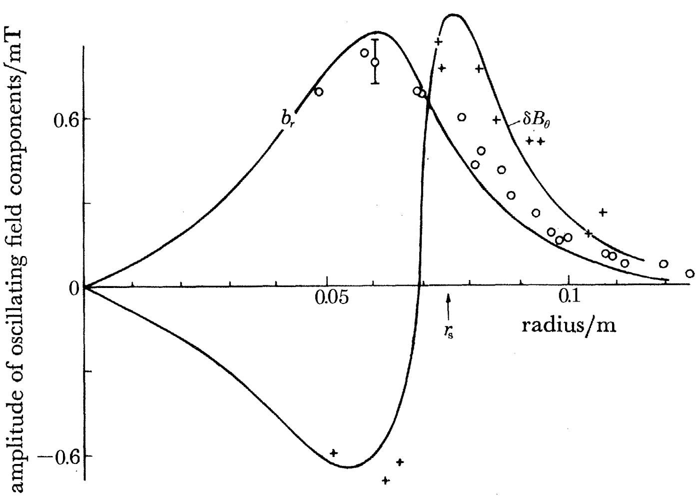

FIGURE 6. A comparison of the measured radial distribution of magnetic field perturbations b_r, δB_θ observed in the small tokamak Tosca with those predicted for an $m = 3$ tearing mode by using the measured current distribution and value of S. The radial position at which $q = 3$ is indicated by r_s.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

shows the measured radial distribution of the oscillating radial and poloidal fields for a mode with $m = 3$. The computed distribution for an $m = 3$ tearing mode for the measured value of S and current distribution is also shown.

A third instability which can appear if the operating parameters are not carefully selected is the disruptive instability. The final phase of this instability is an abrupt expansion of the temperature and current profiles, which can terminate the plasma current and damage the walls of the containing vessel. This instability appears to be associated with a growing $m = 2$ tearing mode whose magnetic islands (figure 7) become too large and interact with the outer cool region of the plasma, forcing the current channel to shrink. This may induce other instabilities such as $m = 1$, $n = 1$ and/or $m = 3$, $n = 2$ which may then interact with the growing $m = 2$ mode. Nonlinear calculations predict that a large $m = 2$ tearing mode will drive unstable other tearing modes (Waddell 1979) that have different poloidal mode numbers, m. Many instabilities with different mode numbers are observed experimentally shortly before a disruption.

Further confirmation of this picture has been obtained by artificially creating magnetic islands by using external helical windings. This makes it possible to trigger a disruption (Karger 1974). From the magnitude of the perturbed fields required (*ca.* 2–4 % of the poloidal field) it is possible that the magnetic islands interact and lead to destruction of the magnetic surfaces, forming regions of ergodic or random magnetic field lines. Some of the islands may join together by field line reconnection which leads to a rapid expansion of the electron temperature profile. The detailed physics underlying these processes is not yet understood and has not yet been fully explored experimentally.

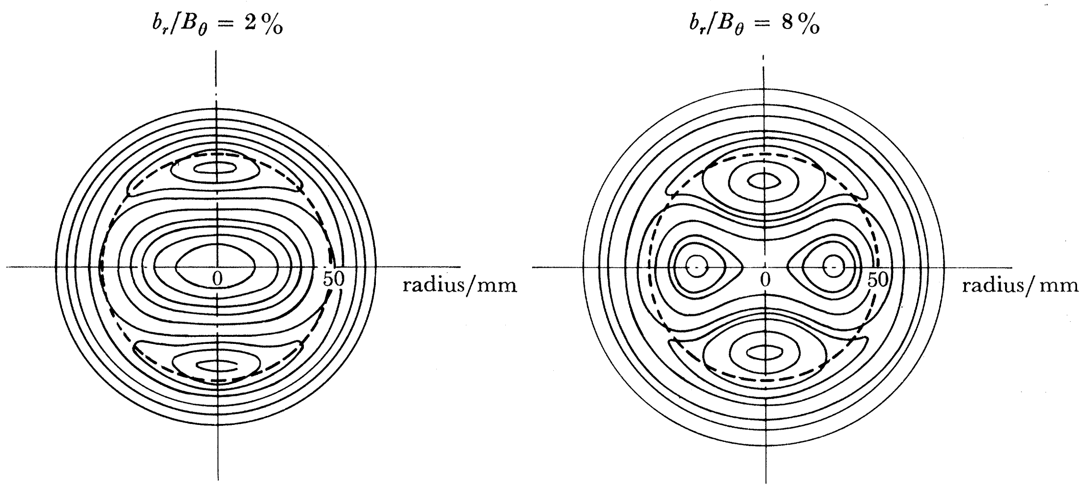

FIGURE 7. The growth of $m = 2$ magnetic islands obtained from magnetic measurements in the small tokamak Tosca up to the onset time of a major disruption when the perturbation level, b_r/B_θ, reaches 4–8 %.

Relaxation and reconnection

Experiments on a slow time-scale and nonlinear calculations show self-control mechanisms that enable the current distributions to remain close to those indicated in figure 3. Even fast pinch experiments quickly relax towards a state of reduced magnetic energy and in so doing can spontaneously produce a reversed axial field, probably by a process of reconnection of the magnetic field. Wells (1969) has shown that the four constraint integrals (Woltjer 1959) can be used to find stable hydromagnetic states and that the fewer the constraints the lower the magnetic energy in the final state. Taylor (1974) considered that the infinity of constraints associated with a perfectly conducting plasma could be reduced to only two, if lines of force could break and reconnect at all radii, possibly due to resistivity. An appropriate invariant for each magnetic surface is $K \equiv \int \boldsymbol{A} \cdot \boldsymbol{B}\, \mathrm{d}\tau$, where $\boldsymbol{A}$ is the vector potential. The topological changes can preserve the sum of all the K provided that the resistivity is small. A single invariant $K_0 = \int_V \boldsymbol{A} \cdot \boldsymbol{B}\, \mathrm{d}\tau$ then characterizes the system and gives a unique force-free minimum energy configuration characterized by the trapped toroidal flux and K_0. Several features of this theory are borne out by pinch experiments and also by tokamaks at the time of disruption. In many situations the relaxation may not be complete, particularly if only a small number of low-amplitude resistive instabilities are excited, if a significant parallel flow velocity of plasma exists or if the resistivity varies in space and time.

Relaxation and the reconnection of magnetic field lines is an essential feature in the current interpretation of the so-called sawtooth oscillation or internal disruption in a tokamak. Kadomtsev (1977) has shown that a similar invariant exists, and that for 'free' reconnection, i.e. helical perturbations excited at all radii, the final configuration is one with a uniform current density. This corresponds closely to the configuration that follows a major disruption brought about by the interaction of many instabilities of different mode number or helicity for a short time.

These theories illustrate the general tendency for magnetic configurations to relax by a variety of mechanisms which involve field line reconnection. Nevertheless, in tokamaks, 'stable' distributions are observed to be maintained for times up to 1 s, which are far from a state of minimum magnetic energy. The reconnection problems considered in astrophysics (Parker 1980) shows similar features.

Conclusions

The experimental behaviour of plasma instabilities in both pinches and tokamaks is in good agreement with the detailed predictions of ideal and resistive m.h.d. theory. The general tendency for such plasmas to exhibit self-control mechanisms can be understood in general terms; however, rapid reconnection phenomena, as in astrophysics, are less well understood. Studies of the nonlinear development of m.h.d. instabilities have been quite successful in explaining a number of experimental phenomena in pinches and tokamaks when only one or two instabilities are involved. There are uncertainties, however, over a variety of resistive instability phenomena associated with finite plasma pressure.

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